A Wavelet Analysis of the Stock Price-Volume Relationship

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Abstract: The paper uses wavelet multi-resolution decomposition of the stock prices and volume in order to test for Granger causality at different time-scales. The results show that at finer (short-run) time-scales there is a feedback mechanism between the stock prices and volume, while at large (long-run) time-scales the volume Granger causes prices. Thus, the paper offers a reconciliation of the long-standing debate on the direction of causation between the stock prices and volume.

1. Introduction

A long-lasting debate in financial economics literature concerns the direction of causality between stock prices and volume. The debate centers on two well-known Wall Street beliefs: (a) it takes volume to make prices and (b) volume is relatively heavy in bull markets. The former implies that volume causes returns while the latter implies opposite. Prior empirical research used Granger causality tests and obtained conflicting evidence. This paper shows that the debate can be resolved using appropriate analysis methods. Financial markets form dynamical systems that are characterized by the decentralized interactions of large number of participants. In this dynamic environment each agent operates on several time-scales at once, thus not only current events but also past events as well as the agent's expectations of the distant future affect current behavior. Traders who take a very long view (years), traders who take a much shorter view (a few months to a year), and traders for whom minutes are too long participate in financial markets. The traders with long view concentrate on market fundamentals while the traders with short view are interested in short-term deviations. The Granger causality tests that work in other cases will have limited success and perform best only as short-term approximations when applied to return and volume data arising from complex dynamical financial markets with overlaying decisions of various time-scales.

This paper uses wavelets to decompose stock returns and volume into their corresponding time-scale components. The Granger causality tests applied to each scale level helped to resolve the long-lasting debate. The paper uses weekly data on the Istanbul

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Stock Exchange (ISE) for the period 1987-2002. The data are obtained from the ISE on-line system. Although there are several ISE indexes, only the ISE-100 is examined in the paper. The main result of the paper is that the direction of causality depends on the level of the time-scale in a systematic way. At finer time-scales we found a feedback between volume and returns. At moderate time-scales the evidence indicates that returns Granger cause volume. However, volume Granger causes returns (prices) at large time-scales. We also found that the direction of causality is ambiguous over all time-scales. Thus, our approach can also explain the results obtained in the prior literature.

There have been a large number of empirical papers that provide indirect evidence on the relationship between trading volume and stock prices (returns). It is well documented that returns on the New York Stock Exchange (NYSE) tend to follow a U-shaped pattern during the trading day (Harris 1986, 1989; McInish and Wood, 1985, 1990; and Wood, McInish and Ord, 1985). Similar results have also been reported for the Hong Kong Stock Exchange (Ho and Cheung, 1991), the London Stock Exchange (Yadav and Pope, 1992), the Tokyo Stock Exchange (Chang et al., 1993) and the Toronto Stock Exchange (McInish and Wood ,1990b). Jain and Joh (1988), Wei (1992) and Wood, McInish and Ord (1985) show that trading volume also follows a U-shaped pattern during the trading day. Hence, considering the similar patterns observed for volume and returns, a positive correlation between the returns (prices) and trading volume is implied. Harris (1987) offers further support. He finds a positive and significant correlation between changes in volume and changes in squared returns for individual NYSE stocks. Cornell (1981) also finds a positive correlation between changes in volume and changes in absolute price in various futures market contracts. Bessembinder and Seguin (1992) find evidence that a positive relationship between volume and volatility exist. Schwert (1990) claims that volume leads price changes because price changes are an important input into trading strategies. Informed traders will transact when new information (public or private) becomes available. Sine trading based on private information is difficult document, trading volume has generally been examined in the context of public information. Woodruff and Senchack (1988) document that a high level of volume immediately follows earnings announcements. Results in this direction have also been reported by Brown, Clinch and Foster (1992), Cready and Mynatt (1991), Kiger (1972) and Morse (1981). However, Peterson (1990) reports some evidence of a different pattern in put option returns. Barclay and Litzenberger (1988) find evidence that announcements of new equity issues are correlated with large price declines and an abnormally high level of volume. In contrast, Jain (1988) finds that while S&P 500 Index returns respond rapidly to macro-economic news announcements such as the money supply, consumer price index, industrial production and unemployment statistics, trading volume does not respond to these announcements. Thus, Jain's findings imply that there is no direct relationship between trading volume and returns. Karpoff (1987) reviews the previous literature and concludes that volume and changes in absolute returns are positively correlated, but that this correlation is weak for shorter measurement intervals. Karpoff also argues that there is only weak evidence supporting a relationship between volume and price change *per se*.

The analysis in this paper focuses on the causal relationship between stock price and volume. The knowledge of causality between stock prices and volume can be used to

improves short-run forecast of current and future movements in trading volume based on the past stock price movements, and vice versa. This paper uses a new methodology based on wavelet decomposition of time series data to examine the causal relationship between stock prices and volume. We decompose price and volume series into different time-scale components and investigate the direction of casualty at each time-scale. The Granger non-causality testing procedure is used for testing causal relationship at each time-scale. This procedure allows us to uncover causal relationships at various time-scales. The advantage of the procedure lies in its ability to test for causality at different time horizons.

The rest of the paper is organized as follows. We review wavelets and multi-resolution analysis, which allows decomposing a series into different time-scales, in Section 2. The data used in the study is described in Section 3. We present time-scale decomposition and Granger causality tests in Section 4. Section 5 concludes the paper.

2. Wavelets and time-scale decomposition

It has long been recognized that economic decision-making is dependent on the time scale involved, and economists clearly discern between long-run and short-run behavior. Unfortunately, commonly used empirical time series methods are not usually capable of discriminating between short- and long-run behaviors of the time series data involved. Although, an ad hoc segregation between short- and long-run behavior can be made in some analyses, this is far from complete. Wavelets offer the possibility of going beyond this simplifying dichotomy by decomposing a time series into several layers of orthogonal sequences of scales using Mallat's multiscale analysis. Once the series are decomposed into different time-scale components, these scales can then be analyzed individually and compared across different series. For instance, Ramsey and Lampart (1998) use this method to examine the relationship between the money supply and output. The related literature has produced ambiguous and contradictory results regarding the Granger causality of the money supply and output. This has sometimes been attributed to structural breaks and possible non-linearities. Ramsey and Lampart show that the contradictions may well be explained by the existence of overlaying timescale-structured relationships. To unmask these relationships, the authors use a wavelet transform to decompose the time series into a low-frequency base scale that captures the long-run trend and six higher-frequency levels. Applying causality tests to the decomposed series, the authors find that, at the lowest timescales, income Granger causes money, but at (shorter) business-cycle periods, money Granger causes income. At the highest scales, the Granger causality goes in both directions, suggesting a feedback mechanism between the money supply and output. This paper uses the approach in Ramsey and Lampart (1998) in order to investigate the causal relationship between the stock prices and volume at different time-scales.

Wavelets are fundamental building block functions, analogous to the trigonometric sine and cosine functions. Although there is some similarity between Fourier transform and wavelet transform of a time series, Fourier transform extracts details from the signal frequency, but all information about the location of a particular frequency within the signal is lost. In contrast wavelets are localized in time and the signals are examined using widely varying levels of focus. This feature of wavelet transforms makes multi-resolution analysis particularly appealing for this study. Previously, continuous wavelet transform has been

used for analyzing economic time series data (see Gencay et al. 2001 and Balcilar 2003 for a review). In this article, we work with non-decimated (discrete) wavelet transform rather than continuous wavelet transform, because from a statistical point of view, they are well adapted (i.e. search for correlations or noise reduction) and offer a very flexible tool for analysis of discrete time series such as the ones under study here. The advantages of non-decimated wavelet transform also include (1) a much better temporal resolution at coarser scales than with ordinary discrete wavelet transform, and (2) it allows us to isolate time series of the major components of stock prices and volume in a direct way. These components are captured by the wavelet transform through the analysis at different resolutions (time-scales). With the multi-resolution technique, a full set of time-scale components are generated for all time series. Consequently, corresponding time-scale components from stock price and volume data can directly be compared. Here we provide a brief summary of the non-decimated wavelet transform, wavelet spectrum and time-scale decomposition. Daubechies (1992) may be consulted for a more in-depth discussion.

The dilation (scaling) and translation (shift) of one mother wavelet $\psi(t)$ generates the *wavelet* $\psi_{j,k}(t) = 2^{j/2}\psi(2^j t-k)$, where j, $k \in Z$. The dilation parameter j controls how large the wavelet is, and the translation parameter k controls how the wavelet is shifted along the t-axis. The father wavelets $\phi(t)$, which generates the wavelet $\phi_{j,k}(t) = 2^{j/2}\phi(2^j t-k)$, are used to capture the sooth, low-frequency (long-run or large-scale) nature of the data, whereas the mother wavelets are used to capture the detailed, and high-frequency (short-run or small-scale) nature of the data. The father and mother wavelets satisfy

$$\int \phi(t) = 1 \quad \text{and} \quad \int \psi(t) = 0$$

respectively. For a suitably chosen mother wavelet $\psi(t)$, the set $\{\psi_{j,k}\}_{j,k}$ provides an orthogonal basis, and the the orthogonal wavelet decomposition of a time series x_t, for $t = 1,\ldots, n$ is

(1) $$x_t = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_{j=1}^{J}\sum_k d_{j,k}\psi_{j,k}(t)$$

where the maximum scale J is determined by the number of data, k ranges from 1 to the number of coefficients associated with the specified component, the coefficients $s_{J,k}$ represent the lowest frequency smooth components, and the coefficients $d_{j,k}$ deliver information about the behavior of the time series x_t concentrating on effects of scale around 2^{-j} near time $k2^{-j}$. In Eq. (1), the wavelet functions $\phi_{J,k}(t)$, $\psi_{J,k}(t)$, $\psi_{J-1,k}(t),\ldots$, $\psi_{1,k}(t)$ are generated from the father wavelet $\phi(t)$, and the mother wavelet, $\psi(t)$, by *translation* and *scaling* as described above. This wavelet expansion of a time series is closely related to the *discrete wavelet transform* (DWT) of a signal observed at discrete points in time. In practice, the length of the signal, say n, is infinite and, for our study, the data used are at weekly frequency, i.e. the time series x_t in Eq. (1) is a vector $x = (x_{t1},\ldots, x_{tn})$ with $t_i =i/n$ and i=1,…, n. With these notations, the DWT of a vector x is simply a matrix product $d = Wx$, where d is an $n \times 1$ vector of discrete wavelet coefficients indexed by 2 integers, $d_{j,k}$, and W is an orthogonal $n \times n$ matrix associated with the wavelet basis. The DWT is quickly computed through an efficient algorithm developed by Mallat (1989). For computational

reasons, it is simpler to perform the wavelet transform on time series of dyadic (power of 2) length (i.e., $n = 128$, 256, or 512 weeks) in this paper. The DWT of the data x_t are the coefficients $s_{J,k}$ and $d_{j,k}$ for $j=J,J-1,\ldots,1$, im Eq. (1). These are given by

(2) $$s_{J,k} = n^{-1/2} \sum_{t=1}^{n} x_t \phi_{J,k}(t), \quad d_{j,k} = n^{-1/2} \sum_{t=1}^{n} x_t \psi_{J,k}(t) \quad j = J, J-1,\ldots,1$$

The magnitudes of the coefficients measure the importance of the corresponding wavelet term in describing the behavior of x_t. The $s_{J,k}$ are called the smooth coefficients because they represent the smooth behavior of the data. The $d_{j,k}$ are called the detail coefficients because they capture the finer or high-frequency behavior of the data.

One particular problem with DWT is that, unlike the discrete Fourier transform, it is not translation invariant. This can lead to Gibbs-type phenomena and other artifacts in the reconstruction of a function. In order to avoid issues that arise for the DWT this paper uses the non-decimated wavelet transform (NWT). The NWT of the data $(x_{t1},\ldots, x_{tn})$ at equally spaced points $t_i = i/n$ and $i=1,\ldots, n$ is defined as the set of all DWT's formed from all n possible shifts of the data by amounts i/n and $i=1,\ldots, n$. Therefore, unlike the DWT, there are 2^j coefficients on the jth resolution level, there are n equally spaced wavelet coefficients in the NWT:

(3) $$d_{j,k} = n^{-1} \sum_{t=1}^{n} x_t \psi_{J,k}[2^j(i/n - k/n)] \quad k = 0,,\ldots,n-1$$

on each resolution level $j=J,J-1,\ldots, 1$. This will give $\log_2(n)$ coefficients at each location. As an immediate consequence, the NWT becomes translation invariant. Due to its structure, the NWT implies a >ner sampling rate at all levels and thus provides a better exploratory tool for analyzing changes in the scale (frequency) behavior of the underlying data in time.

Another way of viewing the result of a NWT is to represent the temporal evolution of the data at a given scale. This type of representation is very useful for the analysis in this paper, which allows us to compare the temporal variation between time series of stock prices and volume at a given scale. To obtain such results, the smooth signal S_J and the detail signals D_j are defined as follows

(4) $$S_J = \sum_k s_{J,k}\phi_{J,k}(t), \quad D_J = \sum_k d_{J,k}\psi_{J,k}(t),$$
$$D_j = \sum_k d_{j-1,k}\psi_{j-1,k}(t) \quad j = J-1,\ldots,1$$

Given the coefficients in Eq. (4) the wavelet series approximation of the original time series x_t is given by the sum of the smooth signal $S_J(t)$ and the detail signals $D_J(t)$, $D_{J-1}(t)$, $\ldots$, $D_1(t)$ (we will drop the temporal index (t) as in $S_J(t)$ depending on the context):

(5) $$x_t = S_J + D_J + D_{J-1} + \cdots + D_1$$

Note that the orthogonal signal components $S_J(t)$, $D_J(t)$, $D_{J-1}(t)$, $\ldots$, $D_1(t)$ are listed in the order of increasingly finer scale components. Small supports are used to capture signal

variations at fine scales and large supports are used to capture signal variations at high scales. Sequentially, the temporal multi-resolution decomposition of a time series at increasing level of refinements is derived from $D_j(t) = S_j(t) - S_{j-1}(t)$. The fine scale variations are captured mainly by the fine scale detail components D_1 and D_2. The coarse scale components S_J, D_J, and D_{J-1} correspond to lower frequency variations of the signal. In the following analysis we will set $J=1$. Therefore the stock prices and volume are decomposed into six components at increasingly finer details.

3. Data

Our data set contains weekly stock price indices and trading volume series for the ISE. Data are obtained from ISE and denominated in Turkish Lira. Our sample covers the period of 01 January 1988-22 August 2003. The ISE stock price data is based on the ISE-100 index. This index is used as a main indicator of the national market. The ISE-100 index, which we will denote by $P(t)$, which has been calculated since the inception of the ISE, is composed of national market companies except investment trusts. The constituents of the ISE-100 index are selected on the basis of pre-determined criteria directed for the companies to be included in the indices as well as in consideration of their ability to represent relevant sectors, and are also subject to short-selling. A time series plot of the logarithm of the ISE-100 data is given in Figure 1(a). The volume series used in the paper is the raw volume, denoted by $V(t)$, which is the number of shares traded. A time series plot of the logarithm of the volume series is given in Figure 1(b).

4. Multi-resolution analysis and Granger causality tests

The first step in our analysis is to decompose the stock price and volume series into various time-scale components using multi-resolution analysis. The wavelet multi-resolution analysis is performed using the Waveslim package written by Whitcher (2002) for the statistical software package R (Gentleman and Ihaka 2000). There are several wavelet filters one can chose for the wavelet transform. In this paper, the Daubechies orthonormal compactly supported wavelet of length $L=8$ (LA8), (Daubechies 1992), of least asymmetric family is used. However, the qualitative results of the paper do not change when other wavelet filters are used. Following Ramsey and Lamport (1998) we use boundary option infinite since the stock price and volume series clearly contain a trend component. Another choice we have to make is to number of scales, J, used in the analysis. For the length of data at hand $J=5$ is chosen, although the qualitative results do not change with choices of J around 5.

The chosen LA8 wavelet is used to decompose the stock price and volume series into five different sets of orthogonal detail components along with a set of orthogonal smooth component. These components represent the behavior of the data at different time scales. The detail components are denoted by $D5$, $D4$, $D3$, $D2$, and $D1$ in order of increasing frequency intensities or fine scales. The smooth component $S5$ represents the long run trend in the data. Our data is in weekly frequency and the time scales of components S_j or D_j are the time period of 2^j to 2^{j+1} weeks.

The smooth and detail components obtained from the NWT multi-resolution analysis is given in Figure 2 and 3 for the stock piece and volume series, respectively. The features of

the components of each series at each scale might be interest on our own and they have quite interesting features, but our purpose here is to use each pair of components for the same time-scale to test for causal relation. The smooth components $S5$ capture the upward trend in both series. The high oscillations in both price and volume series are reflected in the fine detail components $D1$ and $D2$. It is interesting that at time-scales from 8 weeks to 32 weeks 64 weeks there are quite regular cycles in the data. Although there are other aspects of the time-scale decompositions of the stock price and volume data, they are not direct interest in
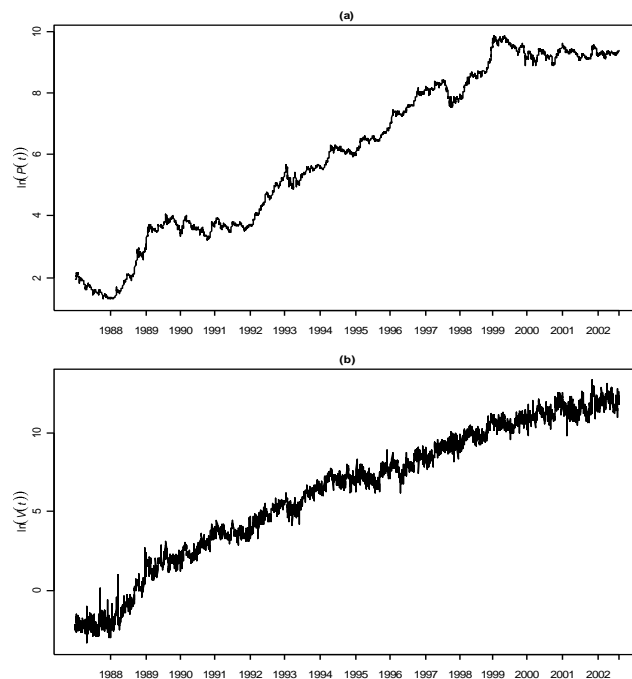
this study.

Figure 1: (a) Logarithm of the ISE-100 index. (b) Logarithm of the ISE volume series.

Our major is interest is the causal relationship between the stock price and volume data at various time scales. The issue of whether the price causes volume or the volume causes price will be examined using the Granger causality tests. The Granger causality tests are joint F tests on the vector autoregressive (VAR) model estimated for the bivariate system compromised of stock price and volume series. Since the direction of the causality at different time-scales is the direct interest in this paper, the direction of causality between the

stock price and volume series is examined by restricting the tests to each specific time-scale. The following VAR model is estimated for each time-scale pairs:

$$(6) \quad \begin{bmatrix} P[\cdot] \\ V[\cdot] \end{bmatrix}_t = \boldsymbol{\alpha} + \begin{bmatrix} \boldsymbol{\beta}_V \\ \boldsymbol{\beta}_P \end{bmatrix} \begin{bmatrix} V[\cdot] \\ P[\cdot] \end{bmatrix}_t + \boldsymbol{\varepsilon}_t, \quad j = J-1,\ldots,1$$

where $P[\cdot]$ and $V[\cdot]$ are the stock price and volume series at each scale, D5, D4, D3, D2, D1, and S5, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}_V$, and $\boldsymbol{\beta}_P$ are vectors of parameters, and $\boldsymbol{\varepsilon}_t$ is the vector of white noise disturbances. We chose the lag length of the VAR model using the Akaike information criteria (AIC). The pairs of two restrictions are tested using the F restrictions. The nulls of the restrictions are

$$H_0^V : \boldsymbol{\beta}_V = 0 \quad \text{and} \quad H_0^P : \boldsymbol{\beta}_P = 0$$

If the null hypothesis H_0^V is rejected, then the stock prices Granger cause the volume. Analogously, the rejection of the null hypothesis H_0^P implies that the volume Granger causes stock prices. For unique decision on the direction of Granger causality when one of the hypotheses is rejected, the other should not be. If both hypotheses are rejected then there exist a feedback relationship between the stock prices and volume. On the other hand both hypothesis are not rejected then the Granger causality test is inconclusive.

The results of Granger causality tests are given in Table 1 along with the chosen lag lengths and p-values. At finer time-scales (D1-D2) the Granger causality tests indicate a either feedback between the stock prices and volume or no causal relationship. At moderate time-scales (D3-D5) the evidence indicates that the Granger causality runs both ways and there is always a feedback between the stock prices and volume. However, volume Granger causes returns (prices) at large time-scales. Thus our findings are consistent with theoretical expectations; in the long- run the market forces certainly play their role and the volume changes lead to price changes. However, in the short-run the financial markets respond to many temporal changes and causality may run in both directions. Thus, our approach can also explain the results obtained in the prior literature.

Timescale	H_0^V	H_0^P	Lag Length	Decision
D1	0.00976	0.02465	135	feedback
D2	0.07101	0.06525	142	inconclusive
D3	0.00001	0.00040	148	feedback
D4	0.0006	0.00005	125	feedback
D5	0.01162	0.00010	150	feedback
S5	0.00015	0.09463	149	$V \rightarrow P$

Table 1: Granger causality tests for stock price and volume series at each time-scale. The lag length is the lag length for each time-scale chosen by the AIC. The p-values for F test are given below each hypothesis. The decision is made at five percent nominal level of significance.
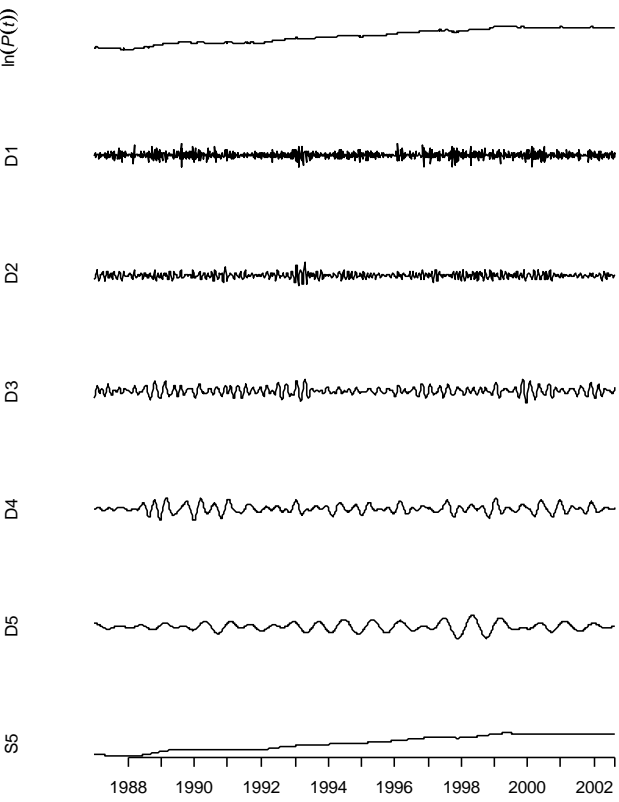
Figure 2: Plots of the individual components of ISE-100 index.
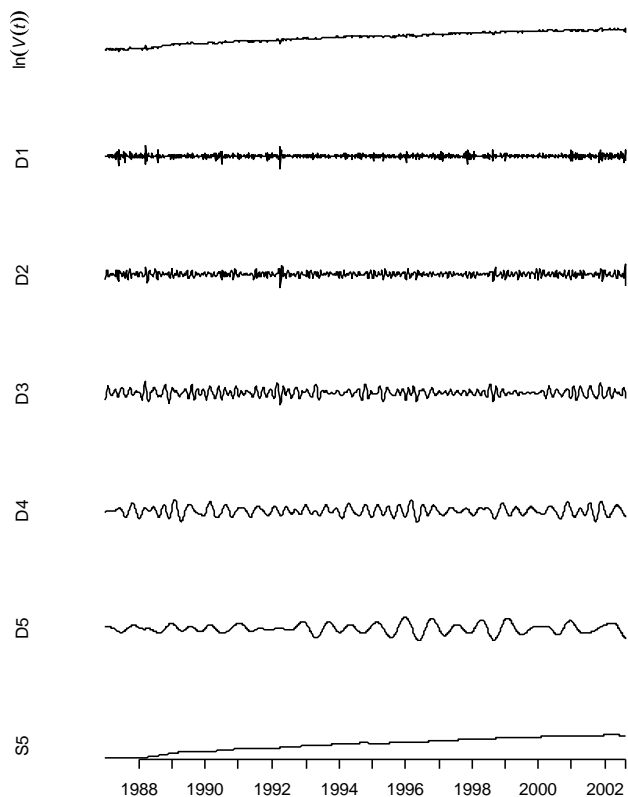The log of the ISE-100 index is included in (a) for comparison.

Figure 3: Plots of the individual components of ISE volume index. The log of the actual volume series is included in (a) for comparison.

5. Conclusion

In this paper we used wavelet multi-resolution decomposition of the stock prices and volume in order to test for Granger causality at different time-scales. There has been a long-lasting debate in the literature on the direction of causality between the stock prices and volume. Our approach reconciles the conflicting evidence using a methodology that takes into account the time-scales. Our results showed that at finer (short-run) time-scales there is a feedback mechanism between the stock prices and volume, while at large (long-run) time-scales the volume Granger causes prices. Thus, the paper offers a reconciliation of the long-standing debate on the direction of causation between the stock prices and volume.

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